Modeling and control of a mobile manipulator in task space

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Abstract— **The present paper shows a systematic approach to modeling and control mobile manipulators that transforms the problem to the modeling of a stationary manipulator stationary with nonholonomic kinematic constraints on the joints. The task-space control consists in an internal compensator of the dynamics of the mobile manipulator and an external proportional–derivative (PD) control with feed-forward of the posture acceleration and an estimate of the derivative of the posture kinematic model. Finally, a numerical experiment is presented using the proposed control and the results are analyzed.**

Keywords: Mobile robots, manipulators, *mobile manipulators*.

I. Introduction

The robots are coming out from the structured environments in factories and they begin to appear in places such as houses, offices and hospitals, where there is little control on the surroundings or not at all (Khatib 1999); the mobile manipulators are a solution for these new workspaces. Basically, a mobile manipulator is a stationary manipulator mounted on a mobile robot so it may perform simultaneously the tasks of locomotion and manipulation; these capabilities give to the mobile manipulator the advantages over stationary manipulators of a bigger task space and a greater autonomy; a mobile manipulator has also disadvantages, such as the presence of nonholonomic kinematic constraints.

Previously, the control of mobile manipulators focused on handling separately the tasks of locomotion and manipulation, for example in Wang *et al.* (2008) the locomotion task is the issue, or in Yu *et al.* (2008) where the manipulation task was the control problem; currently the control problem is to perform both tasks simultaneously, for example in Andaluz *et al.* (2010), where a kinematic control is developed; there are literature that presents results with the dynamic control of a mobile manipulator; in Korayem *et al.* (2010) an optimal dynamic control for a mobile manipulator is developed to track a trajectory that avoids obstacles while considering the maximum load-carrying capacity of the robot. On the other hand, the kinematic modeling of mobile manipulators is still treated as a three-step operation: the modeling of locomotion, the modeling of manipulation and their combination in a global kinematic model; a technique to achieve this operation is in Luca *et al.* (2006), where the kinematic models of locomotion and manipualtion are combined through the so called extended Jacobian, but these models are still obtained by different techniques.

The present paper shows a methodology for the modeling and the control of a mobile manipulator in task space; the mobile manipulator is modeled as an stationary manipulator with kinematic constraints on the joints variables. The outline of this work is as follows: first, a review is presented on modeling techniques for stationary manipulators and mobile robots(Section II). Then, an integrated modeling technique is proposed to obtain the kinematic model of mobile manipulators, transforming the nonholonomic constraints to a mapping between the actuation space and the joint space, using the so called configuration kinematic model (Section III). The resulting kinematic model is applied to obtain a dynamic model of the mobile manipulator (Section IV). Then, a task-space control is developed (Section V.) Finally, the results of numerical simulations for the task-space control are showed assuming a 5-degree of freedom (DOF) differential-traction mobile manipulator (Section VI).

II. Kinematic modeling techniques for stationary and mobile robots

In wheeled mobile robots, the motion is determined by the kinematic constraints of the wheels. A mechanical system is said to be *holonomic* if there is a set of k constraints to the motion; these constraints may be expressed as

$$h_i(q) = 0,\ i = 1, \dots, k \tag{1}$$

where $q(t) \in \mathbb{R}^n$ is the generalized-coordinates vector of the mechanical system and $h_i(q)$ are scalar functions; such constraints are geometric and they limit where may be the system configuration. A mechanical system is called *nonholonomic* if its motion is limited by constraints expressed as

$$a_i(q, \dot{q}) = 0 \tag{2}$$

where $\dot{q}(t) \in \mathbb{R}^n$ is the generalized-velocities vector and $a_i(q, \dot{q})$ are scalar functions; these constraints limit how the mechanical system can move, but does not restrict where the system can be.

In a stationary manipulator, the kinematic model describes the relationship between the posture motion of the robot and its joints motion. This model is conceptually obtained through the so called forward kinematics; the forward kinematics is usually expressed as

$$r_m(t) = f_m(q_m(t)) \tag{3}$$

where $r_m(t) \in \mathbb{R}^{p_m}$ is the posture variables vector, $q_m(t) \in \mathbb{R}^{n_m}$ is the joints displacement vector of the manipulator, p_m is the dimension of the task space of the manipulator and n_m is the dimension of q_m, usually called degree of freedom (DOF). The kinematic model of a stationary manipulator with full-actuated independent joints is then obtained through the time derivative of (3)

$$\dot{r}_m(t) = J_m(q_m)\dot{q}_m(t) \tag{4}$$

where $\dot{r}_m \in \mathbb{R}^p_m$ are the posture velocities, $\dot{q}_m \in \mathbb{R}^n_m$ are the joint velocities of the manipulator, and the matrix $J_m(q) \in \mathbb{R}^{p_m \times n_m}$ is the so called Jacobian and it is defined as

$$J_m(q_m) = \frac{\partial f_m}{\partial q_m}(q_m). \tag{5}$$

In mobile robots, there are two kinds of kinematic models (Campion *et al.* 1996). The first one establishes the relation of the posture motion of the final effector and the actuators motion. The posture kinematic model of a wheeled mobile robot with differential traction can be expressed as

$$\dot{r}_b(t) = B(q_b)\eta_b \tag{6}$$

where $\eta_b(t) \in R^{n_b - k}$ is the vector which contains the velocities of the actuators, $q_b \in \mathbb{R}^{n_b}$ is the configuration of the mobile base, n_b and p_b are the dimensions of the mobile base configuration and posture, and $B(q_b) \in R^{p_b \times (n_b - k)}$ is a matrix with its columns are a base of the null space of the nonholonomic constraints; the posture kinematic model is useful in the computation of control laws in the task space; on the other hand, the configuration kinematic model is used to simplify the dynamic model of mobile robot.

The second model describes a relation between the motion of the joint displacements and the motion of the actuators, called configuration kinematic model, and it is defined as

$$\dot{q}_b = S_b(q_b)\eta_b \tag{7}$$

where $S_b(q) \in R^{n_b \times (n_b - k)}$ is a matrix with its columns belongs to the null space of the constraint matrix A_b. For example, in a differential-traction mobile robot the configuration kinematic model can be defined as

$$S_b(q) = \begin{bmatrix} \cos\phi & 0 \\ \sin\phi & 0 \\ 0 & 1 \end{bmatrix}. \tag{8}$$

It is important to remark that $S_b(q)$ is an annihilator of the kinematic constraints, such that

$$A_b(q_b)^T S_b(q_b) = 0 \tag{9}$$

where the matrix $A_b(q) \in \mathbb{R}^{n_b \times k}$ defines the nonholonomic kinematic constraint

$$A_b(q)\dot{q} = 0; \tag{10}$$

this fact could be used to simplify the dynamic model.

III. Kinematic modeling of mobile manipulators

The problem with the current methods of kinematic modeling of mobile manipulators is that they separate the modeling of the mobile base from the modeling of the manipulator arm. As a second step, the kinematic models of the mobile base and the manipulator are obtained; the kinematic model of the mobile base can be derived from the forward kinematics; on the other hand, the geometric Jacobian could be used in the kinematic model of the mobile manipulator. As a third step, these models are united through the extended Jacobian, which inserts the effects of the nonholonomic constraints in the model. The present Section shows these methods and introduces an integrated kinematic modeling technique for mobile manipulators which uses those tools already in use for manipulator arms, such as the Denavitt–Hartenberg parameters and the geometric Jacobian.

As stated before, the forward kinematics of the mobile base and the manipulator arm are determined through different techniques; one way to determine the forward kinematics of a mobile manipulator is to use the homogeneous transformations

$$T^0_n = T^0_b T^b_n \tag{11}$$

where the matrix $T^0_b \in \mathbb{R}^{4\times 4}$ is the homogeneous transformation of the mobile base that goes from a frame $\{b\}$ fixed on the mobile base to a frame $\{0\}$ fixed on the surface on which the mobile base moves, and the matrix $T^b_n \in \mathbb{R}^{4\times 4}$ is the homogeneous transformation of the manipulator arm that goes from a frame $\{n\}$ fixed on the last link of the mobile manipulator to the frame $\{b\}$. The equation (11) indicates how to combine the forward kinematics of both the mobile base and the manipulator arm but does not state how to calculate the required homogeneous transformations T^0_b and T^b_n; the homogeneous transformation of the mobile manipulator,T^b_n, could be calculated through the Denavitt–Hartenberg method; on the other hand, there is not a standard method to find the homogeneous transformation of mobile base, T^0_b, on the reviewed literature.

It is important to remark that the kinematic model of a mobile manipulator must take account of the relationship between the actuation variables and the configuration variables; Usually in a manipulator arm, the relation between the joint variables and the actuation variables is the identity, but in a system with nonholonomic constraints this is not the case. After obtaining the kinematic models, they are combined using, for example, the so called extended Jacobian (Luca *et al.* 2006), which is defined as

$$\dot{r} = \begin{bmatrix} J_b(q_b)S_b(q_b) & J_m(q_m) \end{bmatrix} \eta \tag{12}$$

where $r \in \mathbb{R}^p$ is the posture of the mobile manipulator, with

$$r = \left[\begin{array}{cc} r_b^T & r_m^T \end{array} \right]^T, \tag{13}$$

J_b is the Jacobian of the base, $\dot{r}$ is a vector that combines the posture velocities of the mobile base and the manipulator arm

$$\dot{r} = \left[\begin{array}{cc} \dot{r}_b^T & \dot{r}_m^T \end{array} \right]^T, \tag{14}$$

and $\eta \in \mathbb{R}^m$ are the actuators velocities for the mobile manipulator and are defined as

$$\eta = \left[\begin{array}{cc} \eta_b^T & \dot{q}_m^T \end{array} \right]^T \tag{15}$$

where $\dot{q}_m$ are the velocities of the mounted manipulator and which also correspond to the velocities of the actuators of the manipulator arm.

The configuration kinematic model is a mapping between the actuation space, which is the set of all possible motions of the actuators, and the joint space (Figure 1); those motions are restricted by the kinematics constraints and the mapping is usually an identity matrix in a stationary manipulator. The Jacobian is a mapping between the joint space and the posture space; Finally, the posture kinematic model is the combination of the Jacobian and the configuration kinematic model. Thus the kinematic modeling of a mobile manipulator depends on finding the Jacobian J and this in turn depends on combining the kinematics of the manipulator and the base mobile. The forward kinematics of a mobile manipulator is given by the function f, defined as

$$r = f(q) \tag{16}$$

where r is the combined posture of the mobile manipulator and q are the generalized coordinates of the mobile manipulator, defined as

$$q = \left[\begin{array}{cc} q_b^T & q_m^T \end{array} \right]^T \tag{17}$$

where q_b and q_m are the generalized coordinates of the mobile base and the manipulator arm respectively.

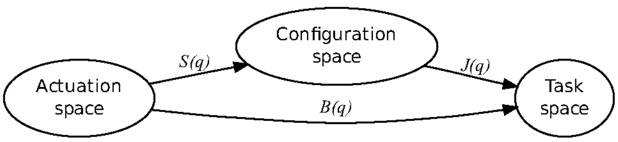

Fig. 1. The posture kinematic model as a composition of mappings between the actuation space, the configuration space and task space.

In the present paper the method proposed is to obtain the forward kinematics of the mobile base T_b^0, by initially assuming that the mobile base is stationary manipulator of b DOF; for example, a differential traction mobile robot could be modeled as link connected to the surface with planar joint, which in turn could be modeled as two prismatic joint and a revolute joint; as a general case, every mobile robot, for example a flying vehicle, could be modeled as an equivalent 6 DOF stationary manipulator. Following this assumption, it is possible to obtain the forward kinematics of the whole mobile manipulator by considering it a single kinematic chain, and then applying a standard modeling method for stationary robots, such as the Denavit–Hartenberg method. Also following the last assumption, it could be possible to obtain the Jacobian J_b from the same geometric method used in stationary robots; even more, it is possible to find the Jacobian of the whole mobile manipulator

$$J(q) = \left[\begin{array}{cc} J_b(q_b) & J_m(q_m) \end{array} \right] \tag{18}$$

with the geometric method, and (12) is reduced to the posture kinematic model of mobile manipulator

$$\dot{r} = B(q)\eta \tag{19}$$

where $B(q)$ is the posture kinematic relation of the mobile manipulator, defined as

$$B(q) = J(q)S(q) \tag{20}$$

$S(q)$ is the configuration kinematic relation for the whole mobile manipulator

$$S(q) = \left[\begin{array}{cc} S_b(q) & I \end{array} \right] \tag{21}$$

where I is a identity matrix, that indicates which configuration velocities are identical to actuation velocities.

The proposed methodology has many advantages, for example it uses the same methods as the stationary manipulators to obtain the forward kinematics and the kinematic models. Another advantage is that the existing computational tools for stationary robots could be used.

IV. Dynamic model of a mobile manipulator

The dynamics of a mechanical system with nonholonomic constraints can be modeled by a set of n second-order differential equations and expressed as follows

$$\begin{aligned} D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) &= S(q)\tau + A(q)\lambda && (22) \\ A(q)^T\dot{q} &= 0 && (23) \end{aligned}$$

where $\tau \in \mathbb{R}^m$ are the generalized forces that go into the system; $D(q) \in \mathbb{R}^{n\times n}$ is the inertia matrix for the mechanical system, $C(q,\dot{q}) \in \mathbb{R}^{n\times n}$ is the Coriolis and cross-velocities matrix, $g(q) \in \mathbb{R}^n$ is a vector which represents the effect of gravity on the links, and $S(q) \in \mathbb{R}^{m\times n}$ in the input matrix. The effect of the kinematics constraints on the dynamics of the mechanical system are expressed on the second term of the right hand side of equation (22) where $A(q) \in \mathbb{R}^{n\times k}$ is a coefficient matrix of a set of k kinematics constrains given in the Pfaffian equation (23), and $\lambda \in \mathbb{R}^k$ is a Lagrange multiplier.

The main problem with equation (22) is the Lagrange multiplier λ, whose value is unknown; taking advantage of the relation (9), it is possible to annihilate the coefficient matrix $A(q)$ in (22) by pre-multiplying by $S(q)^T$ and then the transformation (6) is applied, thus the following expression is obtained

$$\begin{array}{rcl} \dot{q} & = & S(q)\eta \\ \dot{\eta} & = & -M(q)^{-1}m(q,\eta) \\ & & +M(q)^{-1}S(q)^T S(q)\tau \end{array} \quad (24)$$

where

$$\begin{array}{rcl} M(q) & = & S(q)^T D(q) S(q) \\ m(q,\eta) & = & S(q)^T D(q)\dot{S}(q)\eta \\ & & +S(q)^T C(q, S(q)\eta)S(q)\eta \\ & & +S(q)^T g(q) \end{array} \quad (25)$$

It is important to remark that the state dimension in (24) is less than in (22).

V. Robust Lyapunov-based control in task space of a mobile manipulator

The proposed control in this paper is divided in two control loops in cascade. The internal loop control uses an inverse dynamics control. The external control loop is a resolution of acceleration control over the task space.

The inverse dynamics compensator is based on the expression (22) to find a suitable τ such that it cancels the dynamics of a nominal system

$$\tau = \left(S(q)^T S(q)\right)^{-1}\left(\hat{M}(q)a + \hat{m}(q,\eta)\right) \quad (26)$$

where $a(t) \in \mathbb{R}^{n-k}$ is the acceleration reference for the system, $\hat{M}$ and $\hat{m}$ are the nominal values of the matrices M and m. Applying (26) to (22) results in the system

$$\begin{array}{rcl} \dot{q} & = & S(q)\eta \\ \dot{\eta} & = & a + \epsilon_d \end{array} \quad (27)$$

where ϵ_d is the uncertainty of the dynamic model and is defined as

$$\epsilon_d = Ea + M^{-1}\tilde{m}, \quad (28)$$

E is a matrix defined by

$$E = M^{-1}\hat{M} - I, \quad (29)$$

I is the identity matrix and $\tilde{m}$ is

$$\tilde{m} = M^{-1}(\hat{m} - m); \quad (30)$$

the new system (27) can have any other desired control in an external loop.

The relation between the actuators accelerations and task-space acceleration is given by the time derivative of (7)

$$\ddot{r} = B(q)\dot{\eta} + \xi \quad (31)$$

where $\xi \in \mathbb{R}^m$ is defined as

$$\xi = \dot{B}(q)\eta, \quad (32)$$

and $\dot{B}(q)$ is the time derivative of $B(q)$. A problem with (32) is to obtain explicitly $\dot{B}(q)$. In the present paper a method is proposed to estimate numerically the value of such expression, which uses only numerical information about the values of $B(q)$. First, the equation (31) is rewrote as

$$\xi = -B(q)\dot{\eta} + \ddot{r}; \quad (33)$$

then, the definitions of $\ddot{r}$ and $\dot{\eta}$ are applied to (33)

$$\xi = -B(q)\frac{d}{dt}\eta + \frac{d}{dt}\dot{r} \quad (34)$$

and finally the expression (19) to the first term on the right

$$\xi = -B(q)\frac{d}{dt}\eta + \frac{d}{dt}\left(B(q)\eta\right). \quad (35)$$

An estimate of the expression (35) could be founded through Euler approximation of the derivative

$$\hat{\xi} = \frac{1}{h}(B(q(t)) - B(q(t-h)))\eta(t-h). \quad (36)$$

where $h \in \mathbb{R}$ is the sampling period. Then the equation (35) can expressed as

$$\xi = \hat{\xi} + \epsilon_k \quad (37)$$

where ϵ_k is the approximation error and is in the order of $o(h^2)$. The resolved acceleration in the actuators is then defined as

$$a = B(q)^{\dagger}(a_x - \hat{\xi}) \quad (38)$$

where $a_x \in \mathbb{R}^{n-k}$ and $(\cdot)^{\dagger}$ denotes the pseudo-inverse.

For the external control loop, a robust task-space control is used (Spong *et al.* 2006). Firstly, a measure of the error on task space is proposed $\tilde{r}$, such that

$$\tilde{r}(t) = r^d(t) - r(t) \quad (39)$$

where $r^d(t) \in \mathbb{R}^n$ is the desired posture. The proposed control law is

$$a_x = \ddot{r}^d + k_1\dot{\tilde{r}} + k_0\tilde{r} + \delta \quad (40)$$

where a_x is the resulting acceleration on task space, $\dot{\tilde{r}}$ is the time derivative of the error with respect time, k_0 and k_1 are some positive scalar constants; one possible definition of $\delta \in \mathbb{R}^m$ is

$$\delta = \begin{cases} -\rho(e)\frac{G^T Pe}{\|G^T Pe\|} & \text{if } \|G^T Pe\| \neq 0 \\ 0 & \text{if } \|G^T Pe\| = 0 \end{cases} \quad (41)$$

where ρ is a function of the error over the scalars, defined as

$$\rho(e) = \frac{1}{1-\alpha}\left(\gamma_1\|e\| + \gamma_2\|e\|^2 + \gamma_3\right), \quad (42)$$

and α and γ_i are scalars. The stability of the control is showed in Theorem 1.

Theorem 1. The system (24) with the controls in cascade (26), (38) and (40) is stable if δ is defined as (41).

Proof: Let be the system (24) with the controls in cascade (26), (38) and (40). From control (38) he following expression can be obtained

$$a_x = \hat{\xi} + B(q)a \quad (43)$$

and substituting (27) and (37) in (43) the result is

$$a_x = \xi + B(q)\dot{\eta} - \epsilon. \quad (44)$$

where ϵ is defined as

$$\epsilon = \epsilon_k + B(q)\epsilon_d. \tag{45}$$

Applying (44) to (40) the following error dynamic can be obtained

$$\ddot{\tilde{r}} + k_1\dot{\tilde{r}} + k_0\tilde{r} + \delta + \epsilon = 0. \tag{46}$$

The equation (46) can be expressed as state-space equation

$$\dot{e} = Fe + G(\delta + \epsilon) \tag{47}$$

where $e(t)$ is the state of the error, defined as

$$e = \begin{pmatrix} \tilde{r} \\ \dot{\tilde{r}} \end{pmatrix}, \tag{48}$$

the matrix F is constant and defined as

$$F = \begin{pmatrix} 0 & I \\ -k_0 I & -k_1 I \end{pmatrix}, \tag{49}$$

G is the input matrix

$$G = \begin{pmatrix} 0 \\ I \end{pmatrix}. \tag{50}$$

To test the stability of (47) a candidate Lyapunov function is proposed

$$V = e^T P e \tag{51}$$

where P is a positive-definite matrix. The time derivative of (51) is

$$\dot{V} = e^T(F^T P + PF)e + 2e^T PG(\delta + \epsilon). \tag{52}$$

Since k_0 and k_1 are chosen such that the matrix F is Hurwitz, it is possible chose a positive-definite matrix Q such that

$$F^T P + PF = -Q, \tag{53}$$

where P is definite-positive matrix, and (52) results in

$$\dot{V} = -e^T Q e + 2e^T PG(\delta + \epsilon). \tag{54}$$

A control that can ensure that (54) is negative or zero is (41).

VI. Numerical experiments and results

To test the proposed method, the model of a mobile manipulator was obtained; it is assumed that mobile manipulator is composed by a Pioneer 3DX mobile robot and a Cyton manipulator arm with 7 DOF. The Pioneer 3DX is a differential traction mobile robot and only two joint of the Cyton robot were considered, thus the mobile manipulator is modeled as a 5 DOF system.

To obtain the kinematic constraints it is assumed that the mobile manipulator is a unicycle without slipping; also the surface on which the mobile base moves is flat and horizontal. It is also assumed that the manipulator arm is a 2-joint planar robot, and its links are modeled like rods. The model was obtained numerically using the Matlab's robotics toolbox (Corke 1996).

To obtain the forward kinematics, the mobile base is modeled as a 2-joint Cartesian manipulator and a third

TABLE I

The Denavit–Hartenberg parameters for the 5-DOF mobile manipulator. The angles are in radians and the distances in millimeters.

i	α	a [mm]	θ	d [mm]	Kinematic pair
1	$-\pi/2$	0	0	0	prismatic
2	$\pi/2$	0	$-\pi/2$	0	prismatic
3	0	0	0	237	revolute
4	0	150	0	0	revolute
5	0	168	0	0	revolute

revolute joint. From this description the Denavit–Hartenberg parameters can be obtained.

Following the assumption that a mobile manipulator could be modeled as a stationary manipulator, the configuration of the mobile manipulator, $q(t) \in \mathbb{R}^5$, is defined as:

$$q = \begin{bmatrix} d_1 & d_2 & \theta_3 & \theta_4 & \theta_5 \end{bmatrix}^T \tag{55}$$

where d_1, d_2 are the surface coordinates (x, y) of the mobile base, $\theta_3 = \phi$ is the orientation of the mobile base, and θ_4, θ_5 are the joint variables of the manipulator arm.

On the other hand, the kinematic constraint of the 5-DOF mobile manipulator is given by the matrix $A(q) \in \mathbb{R}^{5\times 1}$ and it is defined by the expression

$$A(q) = \begin{bmatrix} \sin q_3 & -\cos q_3 & 0 & 0 & 0 \end{bmatrix}^T. \tag{56}$$

A possible configuration kinematic model that satisfy (56) is the equation

$$\dot{q} = S(q)\eta \tag{57}$$

where $\eta \in \mathbb{R}^4$ are actuation velocities, defined as:

$$\eta = \begin{bmatrix} v & \dot{q}_3 & \dot{q}_4 & \dot{q}_5 \end{bmatrix}^T \tag{58}$$

where $v(t)$ is an scalar which describes the lineal velocity of the mobile robot, and configuration kinematic model $S(q) \in \mathbb{R}^{5\times 4}$ is defined by

$$S(q) = \begin{pmatrix} \cos q_3 & 0 & 0 & 0 \\ \sin q_3 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{59}$$

which satisfy the property of being an annihilator for (56).

The matrices $D(q)$ and $C(q, \dot{q})$ of the system (22) are obtained through the procedure presented in Spong *et al.* (2006); the data required to calculate those matrices appear on Table I and Table II.

The control described in the Section V was applied to a numerical model of the mobile manipulator. The reference is a circular trajectory in task space (Figure 2). It is remarked that the motion is counterclockwise and the initial movements of the robot are in the other direction. The tracking error converges exponentially to zero and it is stable in the time frame of the simulation (Figure 3).

TABLE II

LINK DATA FROM THE MOBILE MANIPULATOR.

i	Length [mm]	Wide [mm]	Height [mm]	Mass [kg]
3	445	393	237	9.0
4	150	50	50	0.1
5	168	50	50	0.1

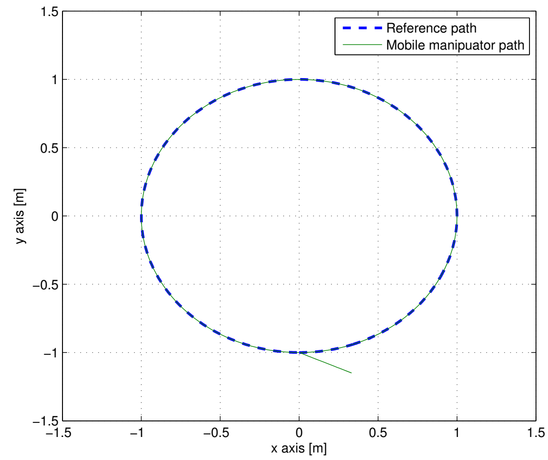

Fig. 2. The reference path and the motion of the robot.

VII. CONCLUSIONS

This paper shows a systematic approach to modeling mobile manipulators that transforms the problem to the modeling of a stationary manipulator stationary with non-holonomic kinematic constraints on the joints; when compared with previous methods, this approach allows to use the same tools as the stationary manipulator and it only requires extending some of the tools in order to handle the kinematic constraints. It is also presented a task-space control that consist in an internal compensator of the dynamics of the mobile manipulator and an external PD control with feed-forward of the posture acceleration and an estimate of the derivative of the posture kinematic model. Finally, a numerical experiment is presented using the proposed control and the results are analyzed.

In future work, it will develop a robust priority control in the task space for a mobile manipulator. Also, it will be developed a robot-aided manipulation system to test these controls.

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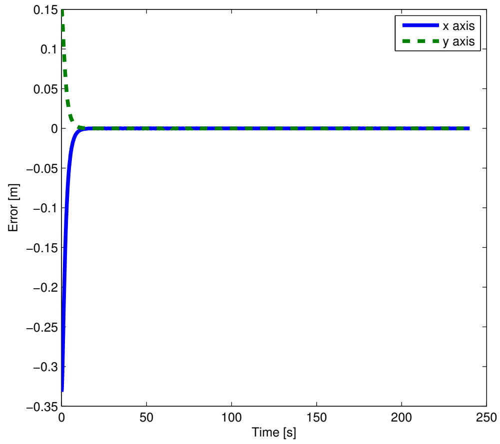

Fig. 3. Posture error graph for the mobile manipulator under the control.